

Digital idle speed control of automotive engines: A safety problem for hybrid systems[☆]

E. De Santis*, M.D. Di Benedetto, G. Pola

University of L'Aquila, Department of Electrical Engineering and Computer Science, Center of Excellence DEWS, Poggio di Roio, 67040 L'Aquila, Italy

Abstract

We address the idle speed control problem in automotive electronics using hybrid methods to derive a digital control law with guaranteed properties. Associating a switching system with the hybrid system that describes the engine operation is crucial to developing a computationally feasible approach. For switching systems with minimum and maximum dwell times and controlled resets, we are able to derive digital control strategies with guaranteed properties that ensure safety. The proposed methodology, while motivated by the idle control problem, is of general interest for hybrid systems for which minimum and maximum dwell times can be established. In our modeling approach, we do not assume synchronization between sampling time and switching time. This is an important technical aspect in general, and in particular for our application, where there is no reason why sampling and switching should be synchronized. Some simulation results are included to demonstrate the effectiveness of the approach.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Idle speed control; Hybrid systems; Switching systems; Safety; Digital control

1. Introduction

Applications of hybrid systems techniques to automotive control have been extensively pursued (see [1,2,7]). These models are required to achieve better control accuracy since the internal combustion engine of a car is intrinsically a hybrid system due to (i) the discrete nature of the four-stroke engine cycle; (ii) the transitions between strokes that are determined by the

[☆] This work was partially supported by the HYCON Network of Excellence, contract number FP6-IST-511368.

* Corresponding author.

E-mail addresses: desantis@ing.univaq.it (E. De Santis), dibenede@ing.univaq.it (M.D. Di Benedetto), pola@ing.univaq.it (G. Pola).

continuous motion of the driveline, which in turn depends on the torque produced by each piston, the actual gear engaged and the connection clutch state. While accurate, the hybrid engine model is complex for control design purposes and may have stability properties that are difficult to analyze. Fortunately, the peculiarities of the engine control problems in each of its working regions (e.g., idle, speed tracking, cut-off) allow simplifications that yield effective control strategies. The idle control problem is one of the most interesting challenges in automotive control and is the motivation for this paper.

The idle speed control problem consists of finding a control strategy that maintains, while in the idle mode, the engine speed in a given range, rejecting torque disturbances due to accessory loads (such as the air-conditioning system and the steering wheel servo-mechanism), and preventing the engine from stalling. This is a typical safety problem where a control strategy has to be studied so that the controlled system never reaches a set of ‘bad’ states. Due to the unpredictable behavior of the torque loads and to the engine operating at very low engine speed, the synthesis of a suitable idle control strategy poses serious challenges to the control designer and makes the idle speed problem hard. We focus on *digital (discrete-time) control laws* since in automotive applications the control strategies are implemented in Engineering Control Units (ECU) that are based on one or more microcontrollers carrying out the computation needed for the control strategies. Traditional techniques used mean-value models (see [4] for a comprehensive list of references related to the idle control problem) that did not capture important transient behaviors. In [3], by using a simplified hybrid model, a control synthesis procedure was proposed based on the computation of the maximal safe set, i.e., the set of all initial conditions starting from which the evolution of the system stays inside the desired range. Once the maximal safe set is computed, following the procedure in [15], it is always possible to derive the *maximal controller*, i.e. the set of all possible control strategies that solve the given problem. However, the resulting control synthesis techniques were still too complex from an industrial point of view, since tuning the controller to different car models required extensive manual recomputation of the control strategy by control engineers.

In [4], a different approach based on a divide and conquer methodology was proposed. First, the overall control system was divided into subparts; then a control strategy for each block in isolation was designed, assuming that the remaining parts can be controlled so that the desired behavior for the whole system was guaranteed. Finally, the correctness of the assumptions made on each subsystem is verified, so that the closed-loop system is guaranteed to behave correctly on the whole.

In [5], starting from a slightly simplified version of the model in [4],¹ we addressed the idle speed control problem from a different perspective: instead of synthesizing a particular control law to be subsequently verified, our methodology was based as in [3] on the computation of the maximal safe set, and carried out using efficient and portable algorithms in the discrete-time domain [9]. However, while this method gave a satisfactory ‘practical’ solution to the problem, the important question of what were the properties of the control strategy when applied to the hybrid system with *continuous* dynamics remained unanswered.

In this paper, we extend the theoretical results obtained in [10] to solve the idle speed control problem, by guaranteeing that the continuous-time dynamics satisfy the constraints. Our strategy is to associate an approximating switching system with the hybrid system under consideration. This is possible in cases where, as in the idle control case, we know that the system ‘dwells’ in

¹ We do not consider the throttle valve dynamics, assuming that it is fast enough to be neglected compared to the manifold and powertrain dynamics.

any given state for at least a given amount of time (*minimum dwell time*) and at most another given amount of time (*maximum dwell time*). Then the transitions of the hybrid system can be considered to occur in a bounded interval of time in an uncontrollable fashion, thus yielding an approximating formulation of the maximal safe set problem for hybrid systems in the classical switching system framework.

In this paper, we extend the results proven in [10] to the class of switching systems with maximum dwell time and controlled reset (a class that includes the idle control problem) to yield a provably correct method for the digital control of these systems under safety specifications. In particular, a constructive procedure is illustrated that allows the computation of a set that is safe for a linear continuous-time switching system \mathcal{S} using a digital controller, on the basis of a safe set computed for a suitable discrete-time switching system associated with \mathcal{S} . Using this procedure, the maximal controlled invariant set for the linear continuous-time switching system can be approximated by an appropriate safe set Ω for the discretized switching system. This approximating set has two important properties: (i) it is safe for the linear continuous-time switching system under digital control, and (ii) if $u'(\cdot)$ is the control law that makes Ω safe for the discrete-time switching system, the piecewise constant control obtained from u' by holding the value at sampling times over the sampling period makes the set Ω safe for the linear continuous-time switching system. We stress that with respect to previous work in this area, *our procedure does not assume that sampling times and switching times are synchronized* and, consequently, that the problem translates into a pure discrete-time problem. We deal directly with the hybrid nature of the problem where event-driven and time-triggered models are mixed. The control strategy obtained with our method is then applied to the idle control problem and its properties for this application analyzed in detail.

The paper is organized as follows. In Section 2, we derive the switching system model of an engine in idle mode and we formalize the digital idle speed control problem. We show that this problem is a special case of the so-called safety problem with digital control. In Section 3, we formally state the digital safety problem. In Section 4, a general procedure for solving safety problems with digital control is developed. In Section 5, the procedure is applied to our case study and the digital idle speed control problem is solved. Finally, Section 6 offers some concluding remarks.

Notations

\mathcal{S}	continuous-time switching system
\mathcal{U}	space of continuous input functions
\mathcal{D}	space of continuous disturbance functions
S_i	linear system associated with the discrete state q_i of \mathcal{S}
T	sampling time
$\mathcal{S}_d(T)$	discretized switching system with sampling time T
$\xi(t, j)$	hybrid state of \mathcal{S}
Λ	hybrid state constraining set for \mathcal{S}
Λ_i	state constraining set for S_i
χ	execution
\mathcal{U}_T	class of T -piecewise constant input functions
δ_M	maximum dwell time
δ_m	minimum dwell time
$\mu(\Omega)$	expanding factor

2. The digital idle speed control problem

The aim of this section is to introduce the formal model of an engine in idle mode, and our control problem. We exploit the peculiarities of the idle region of operation to avoid unnecessary complexity, as was done in [5]. To further simplify the model and improve the tractability of the problem, we then approximate the hybrid system obtained with an appropriate switching system [9] with minimum and maximum dwell times, and controlled reset.

The engine is said to be in *idle mode* if the accelerator pedal is released and no gear is inserted. In this operation region, the car is not moving but the engine should stay ‘alive’. The interesting aspect of this problem is that the revolutions of the engine should stay nearly constant no matter which load is applied to it. It is in general difficult, if not impossible, to forecast when loads such as air conditioning are applied.

The control objective is to find a *digital* control strategy that maintains crankshaft engine speed limited to a range given in terms of nominal speed and maximum absolute tolerance. The motivation for the use of a *digital* controller is that in automotive applications the control strategies are implemented in Engineering Control Units (ECU) that are based on one or more microcontrollers carrying out the computation needed for the control strategies.

The model of an automotive engine in idle mode is now derived.

Continuous dynamics in idle mode

In the digital idle speed control problem two dynamics are of interest: the intake manifold pressure and the crankshaft dynamics. The manifold pressure p , expressed in mbar, is regulated by the throttle opening angle α , expressed in degrees ($^\circ$):

$$\dot{p}(t) = a_p p(t) + b_p \alpha(t), \quad t \geq 0, \quad (1)$$

where $a_p = -20.9440 \text{ s}^{-1}$ and $b_p = 1821.2 \text{ mbar}/^\circ \text{ s}$. We assume that $p(t) \in [p_{\min}, p_{\max}]$, $t \geq 0$, where $p_{\min} = 150 \text{ mbar}$ and $p_{\max} = 250 \text{ mbar}$. The control variable α is limited to a given interval, $\alpha(t) \in [0^\circ, \alpha_{\max}]$, $t \geq 0$, with $\alpha_{\max} = 5^\circ$, in order to avoid manifold pressure rising too much and to limit the control range for safety reasons. Crankshaft variables of interest are the angular position θ_C , expressed in degrees, and the revolution speed n , expressed in RPM (Revolutions Per Minute); the crankshaft angle θ_C evolves according to the following expression:

$$\dot{\theta}_C(t) = K_C n(t), \quad t \geq 0, \quad (2)$$

where $K_C = 6$ is the factor that transforms the RPM into $^\circ/\text{s}$. We assume that $\theta_C \in [0^\circ, 180^\circ]$. The crankshaft speed evolves with the dynamics:

$$\dot{n}(t) = a_n n(t) + b_n (T_g(t) - T_l(t)), \quad t \geq 0, \quad (3)$$

where T_g is the engine generated torque, expressed in N m, and T_l is the disturbance torque, expressed in N m, that models the effect of the loads to the engine coming from subsystems that take energy from the engine and $a_n = -1.5308 \text{ s}^{-1}$ and $b_n = 95.4930 \text{ RPM N m}^{-1} \text{ s}^{-1}$. We assume that $T_g \in [T_{\min}, T_{\max}]$, where $T_{\max} = -T_{\min} = 100 \text{ N m}$, and $T_l \in [0, T_{l \max}]$, where $T_{l \max} = 5 \text{ N m}$.

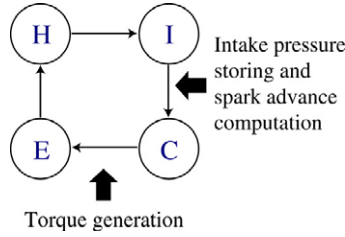


Fig. 1. Single cylinder evolution.

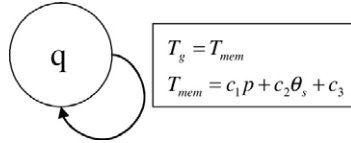


Fig. 2. Torque generation model.

Torque generation

In a four-stroke gasoline engine, torque is generated by a piston when it reaches the highest position in the cylinder and the air–fuel mix entrapped is ignited. In the model, torque is assumed constant during the entire expansion stroke. The torque generation mechanism and the stroke evolution² are represented by the FSM in Fig. 1; each transition occurs when the piston reaches one of the dead centers. Engine torque is expressed either with complex polynomials or look-up tables that cover almost every engine speed and manifold pressure range. In our application, since engine speed is limited to a range, we have a limited torque range and therefore we can simplify the model substantially:

$$T(t_{C-E}) = c_1 p(t_{I-C}) + c_2 \theta_s(t_{I-C}) + c_3, \tag{4}$$

where $c_1 = 0.1125 \text{ N m mbar}^{-1}$, $c_2 = -0.5238 \text{ N m} / ^\circ$, $c_3 = -0.4711 \text{ N m}$, and $p(t_{I-C})$ and $\theta_s(t_{I-C})$ are respectively the intake manifold pressure and the spark advance at the end of intake stroke, corresponding to a bottom dead center. We consider the spark advance angle θ_s as the deviation from optimal spark advance, given as a function of the engine working point. The spark advance angle is bounded to avoid knock (too much advance) and misfire (too little advance); we assume that $\theta_s \in [\theta_{s \text{ min}}, \theta_{s \text{ max}}]$, where $\theta_{s \text{ min}} = 0^\circ$ and $\theta_{s \text{ max}} = 20^\circ$. For example, $\theta_s = 0^\circ$ means that spark coils are programmed to provide the spark at the angular position corresponding to the optimal spark advance. In a four-cylinder four-stroke engine only one cylinder can be in any one stroke, so only one cylinder is producing torque. Hence, torque is generated every 180° of crankshaft angular position and the FSM of Fig. 1 reduces to the one shown in Fig. 2, where the transition is taken when a piston reaches the Top Dead Center TDC_{C-E} , (referred to in the sequel simply as *TDC*).

The dynamics presented above are heterogeneous: pressure and angular speed follow continuous-time dynamics, while torque generation is *event-driven*, because of the torque value reset at every *TDC*. These models merge in the hybrid model \mathcal{H}_c , shown in Fig. 3. The hybrid

² We are dealing with four-stroke SI engines.

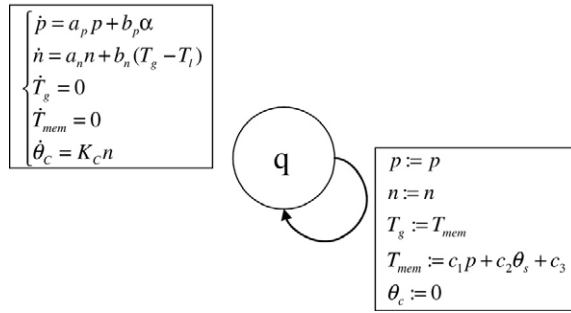


Fig. 3. Hybrid engine model \mathcal{H}_c .

model \mathcal{H}_c is composed of one discrete state characterized by five continuous dynamics. The discrete transition occurs when crankshaft angle θ_C reaches 180° . Whenever a discrete transition occurs, the continuous variables are instantly reset to new values (see Fig. 3).

We approach the problem of controlling the hybrid model \mathcal{H}_c by ‘simplifying’ its semantics in such a way that the behavior of the resulting process contains the original one. We do this by associating with the hybrid model \mathcal{H}_c , whose discrete transitions depend on the continuous state, a suitable switching system \mathcal{S}_c in the sense of [10,9], whose discrete transitions are caused by external discrete disturbances, and which is an outer approximation (abstraction) of the original hybrid system \mathcal{H}_c . Our motivation for focusing on this subclass of hybrid systems comes from the interest in obtaining computationally feasible procedures (see e.g. [9]).

The transition mechanism of a switching system is in general bounded between a lower bound δ_m , called the *minimum dwell time* [16] and an upper bound δ_M , called the *maximum dwell time*, so that the time interval Δ between two consecutive transitions satisfies $\delta_m \leq \Delta \leq \delta_M$.

If the discrete transition of \mathcal{H}_c is replaced by a switching transition with $\delta_m = 0$ and $\delta_M = +\infty$, the resulting switching system approximates \mathcal{H}_c . However, by exploiting the semantics of the discrete transition of \mathcal{H}_c , a tighter approximation can be obtained. In fact, a minimum and a maximum dwell time are naturally induced in the switching system \mathcal{S}_c : given the constraints on the engine speed in idle mode, the distance in time between *TDCs* is limited to an interval. From the engine speed specification $n \in [n_0 - \Delta_n, n_0 + \Delta_n]$, it is possible to find the range $[\delta_m, \delta_M]$ for a *TDC* period, as

$$\delta_m = \frac{180}{K_C(n_0 + \Delta_n)} = 0.0353 \text{ s}, \quad \delta_M = \frac{180}{K_C(n_0 - \Delta_n)} = 0.04 \text{ s}. \tag{5}$$

The digital idle speed control problem can be recast as a safety problem for the switching system \mathcal{S}_c :

Problem 1 (*The Digital Idle Speed Control Problem*). Given the switching system \mathcal{S}_c with minimum and maximum dwell times (5), find a digital control that *maintains crankshaft* engine speed (n) limited to a range, given in terms of nominal speed (n_0) and maximum absolute tolerance (Δ_n):

$$n \in [n_0 - \Delta_n, n_0 + \Delta_n],$$

where $n_0 = 800$ RPM and $\Delta_n = 50$ RPM.

The problem consists in finding, first, the so-called maximal safe set, i.e. the set of all initial states guaranteeing that there exists a control that maintains the evolution of the system in the good set (the ‘safe’ region of the state space). Then, the objective is the design of a *digital* control that maintains the state of the system in the maximal safe set.

Algorithmic procedures for computing approximations of the maximal safe set for continuous-time switching systems with minimum dwell time, no maximum dwell time and uncontrolled reset have been developed in [10]. However, these results cannot be applied directly to our application problem since the model \mathcal{S}_c is characterized *also* by a maximum dwell time and by a controlled reset. In the two following sections, we extend the results of [10] to general continuous-time switching systems. We then apply our theoretical results to solve the digital idle speed control problem in the last section.

3. Switching systems and safety problems

In this section, we formally state the digital control problem of continuous-time switching systems under safety specifications.

The hybrid state ξ of a linear switching system is composed of two components: the discrete state q_i belonging to a finite set Q and the continuous state x belonging to a linear space \mathbb{R}^{n_i} , whose dimension n_i depends on q_i . The evolution of the discrete state q_i is governed by a Finite State Machine (FSM), while the evolution of the continuous state x is given by a linear dynamical system $S(q_i)$ controlled by a continuous input and subject to continuous disturbances, whose matrices depend on the current discrete state q_i . Whenever a discrete transition occurs, the continuous state is instantly reset to a new value. The reset of the continuous state depends on the discrete states before and after the transition, on the continuous state before the transition and on a control u_r that is applied instantaneously whenever a transition is taken. As such, a switching system is a generalization of a switched system (see e.g. [19,14]) where no underlying structure is defined for the discrete transitions and no reset may occur. Formally,

Definition 2. A linear continuous-time switching system \mathcal{S} is a tuple:

$$(\Xi, \Sigma, U, D, S, E, R),$$

where:

- $\Xi = \bigcup_{q_i \in Q} \{q_i\} \times \mathbb{R}^{n_i}$ is the hybrid state space, where $Q = \{q_i, i \in J\}$ is the discrete state space, $J = \{1, 2, \dots, N\}$ and \mathbb{R}^{n_i} is the continuous state space associated with $q_i \in Q$;
- Σ is the finite set of discrete disturbances;
- $U \subset \mathbb{R}^m$ is the continuous input space; we denote as \mathcal{U} the class of all piecewise continuous control functions $u : \mathbb{R} \rightarrow U$;
- $D \subset \mathbb{R}^s$ is the continuous disturbance space; we denote as \mathcal{D} the class of all piecewise continuous disturbance functions $d : \mathbb{R} \rightarrow D$;
- S is a mapping associating with any discrete state $q_h \in Q$, the linear continuous-time dynamical system

$$\dot{x}(t) = A_h x(t) + B_h u(t) + G_h d(t),$$

with $A_h \in \mathbb{R}^{n_h \times n_h}$, $B_h \in \mathbb{R}^{n_h \times m}$, $G_h \in \mathbb{R}^{n_h \times s}$, $u \in \mathcal{U}$, $d \in \mathcal{D}$; for simplicity $S(q_h) = S_h$;

- $E \subset Q \times \Sigma \times Q$ is a collection of discrete transitions;

- $R : E \times \Xi \times U \rightarrow \Xi$ is the reset map such that, given $e = (q_i, \sigma, q_j) \in E$, $\xi = (q_i, x) \in \Xi$ and $u_r \in U$,

$$R(e, \xi, u_r) = (q_j, M_{ij}x + N_{ij}u_r + P_{ij}),$$

where $M_{ij} \in \mathbb{R}^{n_j \times n_i}$, $N_{ij} \in \mathbb{R}^{n_j \times m}$, $P_{ij} \in \mathbb{R}^{n_j}$.

The triple (Q, Σ, E) can be viewed as a FSM having state set Q , input set Σ and transitions defined by E . This FSM characterizes the structure of the discrete transitions. For formally defining the evolution of the class of linear switching systems, we need to introduce the notion of hybrid time basis. As defined in [15], a *hybrid time basis* τ is an infinite or finite sequence of sets $I_j = \{t \in \mathbb{R} : t_j \leq t \leq t'_j\}$, with $t'_j = t_{j+1}$ and $j = 0, 1, \dots$; the time t'_j is called the *switching time*. Let $\text{card}(\tau) = L + 1$. If $L < \infty$, then t'_L can be finite or infinite. We can assume without loss of generality that $t_0 = 0$. Denote by \mathcal{T} the set of all hybrid time bases. The switching system temporal evolution is then defined by means of the (classical) notion of execution [15].

Definition 3. An execution χ of a linear switching system \mathcal{S} is a collection:

$$(\xi_0, \tau, \sigma, u, u_r, d, \xi),$$

where $\xi_0 = (q_0, x_0) \in \Xi$ is the initial (hybrid) state, $\tau \in \mathcal{T}$ is the hybrid time basis, $\sigma : \mathbb{Z} \rightarrow \Sigma$ is the discrete disturbance function, $u \in \mathcal{U}$ and $u_r : \mathbb{R} \rightarrow U$ are the control functions, $d \in \mathcal{D}$ is the continuous disturbance function and $\xi : \mathbb{R} \times \mathbb{N} \rightarrow \Xi$ is the hybrid state evolution, defined as follows:

$$\begin{aligned} \xi(0, 0) &= \xi_0 = (q_0, x_0); \\ \xi(t_{j+1}, j + 1) &= R(e_j, \xi(t'_j, j), u_r(t'_j)); \\ x(t, j) &= x_h(t, t_j, x(t_j, j), u, d), \end{aligned}$$

where $q : \mathbb{N} \rightarrow Q$, such that $e_j = (q(j), \sigma(j), q(j + 1)) \in E$ and $x_h(t, t_j, x(t_j, j), u, d)$ is the (unique) solution at time t of the dynamical system $S_h = S(q(j))$, with initial time t_j , initial condition $x(t_j, j)$, input function $u|_{[t_j, t]}$ and disturbance function $d|_{[t_j, t]}$.

The definition above specifies the temporal evolution of the switching system that can be considered as indexed collections of dynamical systems, each determining the evolution of the system, except at those instants of time in which there is a ‘jump’ between two different dynamical systems. This jump is due to external events, which act as discrete disturbances. The continuous state after a discrete transition is related to the continuous state before a discrete transition through the controlled reset map.

Throughout the paper, we make the following assumption:

Assumption 4 (Minimum and Maximum Dwell Times). Given the switching system \mathcal{S} , there exist $\delta_m > 0$, called the minimum dwell time [16], and $\delta_M > 0$, called the maximum dwell time, such that

$$\delta_m \leq t'_j - t_j < \delta_M, \quad \forall j = 0, 1, \dots, L,$$

for any switching system execution. The value δ_M can be finite or infinite.

If there is a finite maximum dwell time, we assume that the system is alive, according to the following:

Assumption 5. Given the switching system \mathcal{S} , the FSM (Q, Σ, E) is alive [17], i.e. for any $q \in Q$ there exist $\sigma \in \Sigma$ and $q' \in Q$ such that $(q, \sigma, q') \in E$.

The existence of a minimum dwell time is a widely used assumption in the analysis of switching systems (e.g. [16,14] and the references therein), and models the inertia of the system for reacting to an external (discrete) input. The existence of a maximum dwell time is related to the so-called liveness property of the system and is widely used in the context of Discrete Event Systems (e.g. [17]). Moreover, as shown in the previous section for the hybrid model of the engine in idle mode, minimum and maximum dwell time switching systems offer a method for approximating hybrid systems. Suitable values for δ_m and δ_M can be computed by analyzing the semantics of the discrete transitions (see e.g. (5)).

The switching system \mathcal{S} is constrained by

$$\begin{aligned} \xi(t, j) \in \Lambda &= \bigcup_{i \in J} \{q_i\} \times A_i \subset \Xi, \\ \forall t \in I_j, \forall I_j \in \tau, \forall \tau \in \mathcal{T}, \end{aligned} \tag{6}$$

where each set A_i is assumed to be bounded, convex, with the origin 0 belonging to the relative interior of A_i , U is supposed to be bounded and convex and D bounded, convex and with $0 \in D$.

Given \mathcal{S} and an execution χ , let $\eta(t) = \xi(t, j)$, $t \in [t_j, t'_j[$, $j = 0, 1, \dots, L$. We assume that the hybrid state evolution is available for control synthesis: the class $\mathcal{Y} = \{\eta|_{[0,t]}, \eta : \mathbb{R} \rightarrow \Xi, t \geq 0\}$ embeds all the information on the hybrid state evolution available for control purposes.

Definition 6. A (state-feedback) control strategy φ is a pair of functions $\varphi^c : \mathcal{Y} \rightarrow U$ and $\varphi^r : \Xi \rightarrow U$, such that the function defined by $u(t) = \varphi^c(\eta|_{[0,t]})$, $t \geq 0$ belongs to \mathcal{U} . A switching system \mathcal{S} together with a control strategy φ is called a controlled switching system and its executions with $u(t) = \varphi^c(\eta|_{[0,t]})$, $t \geq 0$, and $u_r(t) = \varphi^r(\xi(t, j))$, $t \in]t_j, t'_j]$, $j = 0, 1, \dots, L$, are called controlled executions.

For a switching system \mathcal{S} under constraints (6), the maximal safe set can be formally defined as follows.

Definition 7. A set $\Omega \subset \Lambda$ is safe for \mathcal{S} if there exists a control strategy φ such that for any disturbance $d \in \mathcal{D}$, the constraints (6) are satisfied for any controlled execution of \mathcal{S} with initial state in Ω and disturbance d . We say that φ makes the set Ω safe for system \mathcal{S} . The maximal safe set of \mathcal{S} is the union of all safe subsets of Λ .

To give a precise formulation of the digital control problem, we have to specialize the definitions of controlled execution and safety for continuous-time switching systems to the case where piecewise constant control laws are used. Given $T > 0$, let $\mathcal{U}_T \subset \mathcal{U}$ be the class of T -piecewise constant functions, i.e.

$$\mathcal{U}_T = \{u \in \mathcal{U} : u(t) = u(kT), t \in [kT, (k + 1)T[, k = 0, 1, \dots\}.$$

A function $u \in \mathcal{U}_T$ can be viewed as a digital control and T as a sampling interval. Note that there is no synchronization between sampling times $t = kT$ and switching times t'_j . Therefore, the controlled execution of the switching system under a T -piecewise constant control strategy must be defined taking into account that the information on the hybrid state is available only at the sampling times. Consequently, the controller is aware of the transition between two discrete states with a time delay less than T , if $T \in]0, \delta_m[$. If $T > \delta_m$, more than one switching may

occur between two consecutive sampling instants. Hence, we assume that $T \in]0, \delta_m[$ to avoid loss of information on the discrete state evolution of the continuous-time switching system.

Given a sampling time T , we can now introduce the notion of T -control strategy that is the specialization of the notion of control strategy to the context of digital controls. A T -control strategy is a control strategy that depends on the hybrid state evolution ξ of \mathcal{S} at sampling times $t = kT, k = 0, 1, \dots$, and such that the resulting control functions belong to \mathcal{U}_T . More formally, let $\mathcal{Y}_T = \{\eta_T|_{[0,t]}, \eta_T : \mathbb{R} \rightarrow \Xi, t \geq 0\}$ be the class of restrictions of $\eta_T(t) = \xi(kT, j), \forall t \in [kT, (k + 1)T], k = 0, 1, \dots$

Definition 8. A T -control strategy φ_T is a pair of functions $\varphi_T^c : \mathcal{Y}_T \rightarrow U$ and $\varphi_T^r : \Xi \rightarrow U$, such that the function defined by $u(t) = \varphi_T^c(\eta_T|_{[0,t]})$ belongs to \mathcal{U}_T . An execution of \mathcal{S} with $u(t) = \varphi_T^c(\eta_T|_{[0,t]})$, $t \geq 0$, and $u_r(t) = \varphi_T^r(\eta_T(t))$, $t \geq 0$, is called a T -controlled execution.

Note that, by definition, $u_r \in \mathcal{U}_T$.

We can now formally state the problem we want to address, which consists in controlling a continuous-time switching system by a digital control for ensuring some safety requirements.

Problem 9. Consider a continuous-time switching system \mathcal{S} and let the hybrid state constraining set Λ be given. Find a sampling time T , a set $X_T \subset \Lambda$ and a T -control strategy such that the constraint (6) is satisfied, for any T -controlled execution with initial state in X_T .

A set $X_T = \bigcup_{i \in J} \{q_i\} \times X_{T,i}$ that solves the problem is called T -safe for the continuous-time switching system \mathcal{S} , under constraints (6).

4. Digital control for safety constraints

In this section, we extend the results of [10] to a switching system with minimum and maximum dwell times and controlled reset, and we develop a procedure that solves Problem 9.

The main idea is to discretize the continuous dynamics of the continuous-time switching system \mathcal{S} and to compute a safe set Ω_d for the discrete-time switching system obtained. On the basis of Ω_d , a safe set Ω_c is computed for the original system \mathcal{S} . The main advantage of this approach is in the synthesis of a digital controller ensuring safety for \mathcal{S} : indeed we will show that a digital controller that makes Ω_c safe can be easily obtained from a discrete controller that makes Ω_d safe.

First of all, we characterize safe sets for a switching system \mathcal{S} . Checking whether a set is safe is in general difficult, since executions with an infinite number of switchings have to be considered. The following lemma gives a ‘one-step’ sufficient condition for checking safety. Given a discrete state q_i , J_i denotes the set of the indices of the successors of q_i , i.e. $J_i = \{j : (q_i, \sigma, q_j) \in E, \text{ for some } \sigma \in \Sigma\}$. The symbol $R_{ij}^{-1} \Omega_j$ denotes the set $\{x \in A_i : M_{ij}x \in \Omega_j - N_{ij}U - P_{ij}\}$ and, given some $\Delta > 0$, $\mathcal{I}_i^\Delta(\bigcap_{j \in J_i} R_{ij}^{-1} \Omega_j)$ denotes the set of initial states such that the state of system S_i belongs to the set $\bigcap_{j \in J_i} R_{ij}^{-1} \Omega_j, \forall t \in [0, \Delta]$, for any disturbance $d \in \mathcal{D}$ and for some suitable control strategy.

Lemma 10. A set $\Omega = \bigcup_{i \in J} \{q_i\} \times \Omega_i \subset \Xi$ is safe for the switching system \mathcal{S} if there exists a control strategy such that for any $i \in J$, any disturbance $d \in \mathcal{D}$ and any $x_0 \in \Omega_i$,

- $x_i(t, x_0, d) \in A_i, \forall t \in [0, \delta_m[$,
- $x_i(t, x_0, d) \in \mathcal{I}_i^{\delta_m - t}(\bigcap_{j \in J_i} R_{ij}^{-1} \Omega_j), \forall t \in [\delta_m, \delta_M[$,

where $x_i(t, x_0, d)$ is the closed-loop solution at time t of the system S_i , with initial time 0, initial state x_0 , and disturbance d .

The conditions expressed in Lemma 10 are not necessary. However, they are necessary and sufficient when Ω is the maximal safe set. Safe sets that satisfy the conditions of Lemma 10 are called *s-safe sets*.

The convex closure of a given safe (*s-safe*) set is still safe (*s-safe*), as the following result shows.

Lemma 11. *If $\Omega = \bigcup_{i \in J} \{q_i\} \times \Omega_i \subset \Xi$ is safe (*s-safe*) for a switching system \mathcal{S} , then $\bigcup_{i \in J} \{q_i\} \times \mathcal{C}(\Omega_i)$, where $\mathcal{C}(\Omega_i)$ is the convex closure [13] of Ω_i , is also safe (*s-safe*) for \mathcal{S} .*

The preservation of safety and *s-safety* under convex closure is a straightforward consequence of the properties of the sets involved and of the linearity of the dynamics, and therefore is omitted.

□

The lack of synchronization between the switching times and the sampling times induces some errors that have to be estimated. Given the continuous-time switching system \mathcal{S} and the constraining set Λ , define the function $\rho : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\rho(T) = \max_{i \in J} \rho_i(T), \tag{7}$$

where $\rho_i(T)$ is the radius of the minimal ball in the appropriate norm, centered in $z \in \Lambda_i$, which contains the state evolution in the time interval $[0, T]$ of the continuous-time dynamical system S_i , for any initial state $z \in \Omega$, for any control function in \mathcal{U}_T , and for any disturbance function in \mathcal{D} , i.e.,

$$\rho_i(T) = \max_{z \in \Lambda_i, u \in U, d \in \mathcal{D}, t \in [0, T]} \left\| e^{A_i t} z + \left(\int_0^t e^{A_i(t-\tau)} B_i d\tau \right) u + \left(\int_0^t e^{A_i(t-\tau)} G_i d(\tau) d\tau \right) - z \right\|_{n_i},$$

where $\|x\|_{n_i}$ is the Euclidean norm of x in \mathbb{R}^{n_i} . The function $\rho(T)$ is used as an *overestimated measure* of the error due to the asynchrony of the switching times and of the sampling times. Moreover, with standard norm manipulations, $\rho(T)$ can be bounded above by a rather easily computable function that asymptotically converges to 0 if T goes to 0. This overestimation of the error due to lack of synchronization does not affect the convergence of our approach towards a solution of Problem 9. Better estimations of the error can be obtained, e.g. with an ad hoc analysis of the problem, as shown in Section 5, where the digital idle speed control problem is solved.

We can now define a suitable *discrete-time* switching system associated with the continuous-time switching system \mathcal{S} , that will be the basis upon which a computable algorithmic procedure, solving Problem 9, is developed. Given the continuous-time switching system

$$\mathcal{S} = (\Xi, \Sigma, U, D, S, E, R),$$

and a sampling time $T > 0$, we define the following discrete-time switching system:

$$\mathcal{S}_d(T) = (\Xi, \Sigma, U, D, S_d, E, R_d), \tag{8}$$

where:

- S_d is a map associating with any discrete state q_i of Q the discrete-time linear system $S_i(T)$ obtained as the exponential discretization of S_i , i.e.

$$x(k + 1) = A_i(T)x(k) + B_i(T)u(k) + G_i(T)d(k), k = 0, 1, \dots,$$

where $A_i(T) = e^{A_i T}$, $B_i(T) = \int_0^T e^{A_i(T-t)} B_i dt$ and $G_i(T) = \int_0^T e^{A_i(T-t)} G_i dt$.

- The reset R_d is a point to set mapping, defined by

$$R_d(e, \xi, u_r) = \{q_j\} \times (M_{ij}x + N_{ij}u_r + P_{ij} + \rho(T)\mathcal{B}_j), \tag{9}$$

for any $e = (q_i, \sigma, q_j)$, $\xi = (q_i, x)$, $u_r \in U$, and $x \in \mathbb{R}^{n_i}$, where \mathcal{B}_j is the unit ball in \mathbb{R}^{n_j} .

The obtained discrete-time switching system $S_d(T)$ exhibits a minimum dwell time δ_m^T and a maximum dwell time δ_M^T that are related to the minimum dwell time δ_m and the maximum dwell time δ_M of S , by

$$\delta_m^T := \text{int}(\delta_m/T) - 1, \quad \delta_M^T := \text{int}(\delta_M/T) + 1, \tag{10}$$

where $r \in \mathbb{R}$, $\text{int}(r)$ denotes the integer lower approximation of r , i.e. $\text{int}(r) = \max\{z \in \mathbb{Z} : z \leq r\}$.

The notions of execution, control strategy and controlled execution naturally extend to the case of the discrete-time switching system $S_d(T)$ and are therefore omitted. A hybrid time basis for $S_d(T)$ is defined over \mathbb{N} , whereas a hybrid time basis for S is defined over \mathbb{R} . With a slight abuse of notation, we denote the hybrid state of $S_d(T)$ by $\xi(k, j)$, $k, j \in \mathbb{N}$.

In the following, given a set $\Omega = \bigcup_{i \in J} \{q_i\} \times \Omega_i \subset \Xi$ and $\alpha \in \mathbb{R}$, we write $\alpha\Omega := \bigcup_{i \in J} \{q_i\} \times \alpha\Omega_i$.

By definition of $S_d(T)$, the following result holds.

Proposition 12. *Given $T > 0$, and a safe set Ω for $S_d(T)$, there exists a T -control strategy such that for any T -controlled execution of S with initial hybrid state in Ω , $\xi(t, j) \in \Lambda, \forall t = kT, k = 0, 1, \dots, j = 0, 1, \dots, L$.*

However, a safe set Ω for $S_d(T)$, is not in general safe for S since the hybrid state evolution ξ of S is not guaranteed to satisfy the constraints (6) at the intersampling times $t \in]kT, (k + 1)T[, k = 0, 1, \dots$. On the other hand, since the state constraining sets Λ_i , the input set U and the disturbance set D are bounded, any hybrid state evolution of S remains bounded, in the intersampling. Therefore, there exists a real $\mu \geq 1$ such that $\mu\Lambda$ contains the hybrid state evolutions in the intersampling times. A measure of μ , i.e. of how much to scale the state constraining set Λ , is therefore required. In [6] we solved this problem for the class of linear dynamical system by introducing the notion of *expanding factor*, which is generalized to switching systems as follows.

Definition 13. Given $T > 0$, and a safe set Ω for the discrete-time switching system $S_d(T)$, the expanding factor $\mu(\Omega) \geq 1$ is the minimum scalar value such that, for some T -control strategy, $\xi(t, j) \in \mu(\Omega)\Lambda, \forall t \in I_j, \forall I_j \in \tau$, for any T -controlled execution of S with initial hybrid state in Ω .

Remark 14. By generalizing the results of [6], it is possible to develop a characterization of the expanding factor, and, in the case where the constraining sets are polytopes, efficient procedures for its computation. We omit this simple generalization here for the sake of conciseness.

By Definition 13, a set Ω that is safe for the discrete-time switching system $\mathcal{S}_d(T)$, constrained by

$$\xi(k, j) \in A, \quad \forall k \in I_j, \quad \forall I_j \in \tau, \quad \forall \tau \in \mathcal{T}, \tag{11}$$

is also safe for the continuous-time switching system \mathcal{S} , constrained by

$$\xi(t, j) \in \mu(\Omega) A, \quad \forall t \in I_j, \quad \forall I_j \in \tau, \quad \forall \tau \in \mathcal{T}. \tag{12}$$

Moreover,

Proposition 15. *If $(u'(\cdot), u'_r(\cdot))$ is the control law that makes Ω safe for the discrete-time switching system $\mathcal{S}_d(T)$, the piecewise constant control law $u(t) = u'(k)$, $u_r(t) = u'_r(k)$, $\forall t \in [kT, (k + 1)T[$, makes the set Ω safe for the continuous-time switching system \mathcal{S} , under constraints (12).*

The proof of the result above follows from [1] and therefore is omitted.

As a consequence of Proposition 15, Ω is a T -safe set for the continuous-time switching system \mathcal{S} , under constraints (12). Furthermore, since the constraining sets A , U , and D are bounded and the continuous dynamics and the reset are continuous, the expanding factor associated with the maximal safe set $\Omega^*(T)$ for $\mathcal{S}_d(T)$ approaches 1 as the sampling time T goes to 0. Therefore, the set $\Omega^*(T)$ converges to a T -safe set for the switching system \mathcal{S} constrained by (6), and hence to a solution of Problem 9, as the sampling time T goes to zero. Since the convergence of the expanding factor $\mu(\Omega^*(T))$ to 1 is in general asymptotic, the convergence of $\Omega^*(T)$ to a solution of Problem 9 is asymptotic as well. However, a finite convergence to a solution of Problem 9 can be obtained as follows. Given a sufficiently small $\varepsilon > 0$, the idea is to compute the maximal safe set $\Omega^*(T)$ for the discretized system $\mathcal{S}_d(T)$, constrained by

$$\xi(k, j) \in \frac{1}{1 + \varepsilon} A, \quad \forall k \in I_j, \quad \forall I_j \in \tau, \quad \forall \tau \in \mathcal{T}, \tag{13}$$

instead of (11). Since $\mu(\Omega^*(T))$ asymptotically approaches 1 when T goes to zero, there exists a finite sampling time $T^* > 0$ for which $\mu(\Omega^*(T^*)) \leq 1 + \varepsilon$. By construction, the hybrid state evolution $\xi(t, j)$ of the continuous-time switching system will remain in the set

$$\mu(\Omega^*(T^*)) \left(\frac{1}{1 + \varepsilon} A \right) \subset A,$$

at the intersampling times $t \in]kT, (k + 1)T[$, $k = 0, 1, \dots, j = 0, 1, \dots, L$, and hence, the set $\Omega^*(T^*)$ is a solution to Problem 9.

We summarize the proposed methodology in the following procedure.

Procedure 16 (Computation of T -safe Sets).

1. Select a sampling time T .
2. While $\Omega = \emptyset$, decrease T .
 - a. Compute the discretized system $\mathcal{S}_d(T)$, as defined in (8).
 - b. Compute an s -safe set Ω for $\mathcal{S}_d(T)$.

End While.
3. Compute a control strategy that makes Ω safe for $\mathcal{S}_d(T)$, constrained by (11).
4. Compute the T -control strategy φ_T that makes Ω safe for \mathcal{S} , constrained by (12) (cf. Proposition 15).

- 5. Evaluate the expanding factor $\mu(\Omega)$ (cf. Remark 14).
- 6. Return Ω and φ_T .

Remark 17. The convergence of Procedure 16 relies on the convergence of Step 2.b., i.e. on the convergence of the procedure for the computation of an s -safe set Ω for $\mathcal{S}_d(T)$. No general result exists which ensures the existence of such a set. However, for stabilizable switching systems sufficient conditions are given in [12] for the existence of an s -safe set for $\mathcal{S}_d(T)$. In the following section, we show that for the idle speed control model, an s -safe set Ω for $\mathcal{S}_d(T)$ exists and can be computed in a finite number of steps. The proposed procedure (Algorithm 20) is written for the particular case study but can be rewritten for a general switching system. This is not done here for the sake of notational simplicity.

5. Solving the digital idle speed control problem

This section is devoted to the application of the methodology described in Sections 3 and 4 for solving the digital idle speed control problem. Some preliminary results on this topic can be found in [11].

Problem 1 may be reformulated more precisely as:

Problem 18. Consider the continuous-time switching system \mathcal{S}_c with minimum and maximum dwell times (5). Let $[p \ n \ T_g \ T_{mem} \ \theta_C]'$ be the continuous state and

$$\Lambda = [p_{\min}, p_{\max}] \times [n_0 - \Delta_n, n_0 + \Delta_n] \times [T_{\min}, T_{\max}] \times [T_{\min}, T_{\max}] \times [0^\circ, 180^\circ] \quad (14)$$

be the state constraining set. Find a sampling time T , a set $X_T \subset \Lambda$ and a T -control strategy such that the constraint (14) is satisfied, for any T -controlled execution with initial state in X_T .

Since the throttle valve controller period is typically around some milliseconds, we chose a sampling time of the same order, i.e. $T = 2.5 \times 10^{-3}$ s.

Step 2.a of Procedure 16 requires the definition of a suitable discretization $\mathcal{S}_d(T)$ of \mathcal{S}_c . The dynamics of the discrete-time system $\mathcal{S}_d(T)$, obtained by exponentially discretizing \mathcal{S}_c , are the following³:

$$x(k + 1) = A_d x(k) + B_d u(k) + G_d T_l(k),$$

where $x(k) = [p(k) \ n(k) \ T_g(k) \ T_{mem}(k)]'$, $u(k) = \alpha(k)$ and the dynamical matrices are

$$A_d = \begin{bmatrix} a_{pd} & 0 & 0 & 0 \\ 0 & a_{nd} & b_{nd} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} b_{pd} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad G_d = \begin{bmatrix} 0 \\ -b_{nd} \\ 0 \\ 0 \end{bmatrix}.$$

For computing the reset point to set map, we first have to evaluate the errors due to the non-synchronization between the switching times and the sampling times.

³ Here, the dynamics of the crankshaft angle θ_C is not considered since it is not essential in the control problem under investigation.

Manifold pressure reading error

If the reset occurs exactly at a sampling time, the pressure read is the manifold pressure at the end of intake stroke used in (4). This is not true in general, so the reading error must be estimated. By integrating the continuous pressure dynamics (1) and considering that the control value is constant during intersampling periods, we obtain $p(t) = e^{a_p t} p(0) + \frac{b_p \alpha}{a} (e^{a_p t} - 1)$. The error is given by $\Delta p = p(t) - p(0) = (e^{a_p t} - 1)(p(0) + \frac{b_p \alpha}{a_p})$, and since $a_p < 0$ and $b_p > 0$, the maximum absolute value is given by

$$\Delta p_{\max} = |e^{a_p T} - 1| \max_{p \in [p_{\min}, p_{\max}], \alpha \in [\alpha_{\min}, \alpha_{\max}]} \left| p + \frac{b_p \alpha}{a_p} \right| = 12.7532 \text{ mbar.}$$

Let $I_p = [-\Delta p_{\max}, \Delta p_{\max}]$ be the admissible value set for Δp .

Engine torque reset

There is no reason to believe that the reset occurs exactly at a sampling time. If the reset does not occur at a sampling time, then torque is not detected for a sample period, i.e. the torque value changes but the controller cannot detect this change for an entire sample period. If ΔT_g denotes the torque error, we can compute the engine speed deviation, where ΔT_g may be viewed as a disturbance in the discrete-time dynamics, $n^+ = a_{nd} n + b_{nd}(T_g - T_l - \Delta T_g)$, where $\Delta T_g \in [-\Delta T_{\max}, \Delta T_{\max}]$. The engine speed disturbance Δn takes values in $I_n = [-b_{nd} \Delta T_{\max}, b_{nd} \Delta T_{\max}]$.

The two errors described above are taken into account in the expression for the reset of $S_d(T)$, as follows:

$$\begin{cases} p := p; \\ n := n + \Delta n; \\ T_g := T_{mem}; \\ T_{mem} := c_1(p + \Delta p) + c_2 \theta_s + c_3. \end{cases} \tag{15}$$

Note that Δp and Δn are monotonically decreasing with the sampling period. It is convenient to rewrite the reset map (15) by means of the following equation where the switching time is denoted by t_{TDC} :

$$x(t_{TDC}^+) = M_r x(t_{TDC}^-) + N_r u_r(t_{TDC}^-) + P_r + G_r d_r(t_{TDC}^-), \tag{16}$$

where

$$\begin{aligned} u_r(t_{TDC}^-) &= \theta_s(t_{TDC}^-) \in [\theta_{s \min}, \theta_{s \max}], \\ d_r(t_{TDC}^-) &= \begin{bmatrix} \Delta p(t_{TDC}^-) \\ \Delta n(t_{TDC}^-) \end{bmatrix} \in I_p \times I_n, \end{aligned}$$

and the matrices are

$$M_r = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ c_1 & 0 & 0 & 0 \end{bmatrix}, \quad N_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ c_2 \end{bmatrix}, \quad P_r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ c_3 \end{bmatrix}, \quad G_r = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ c_1 & 0 \end{bmatrix}.$$

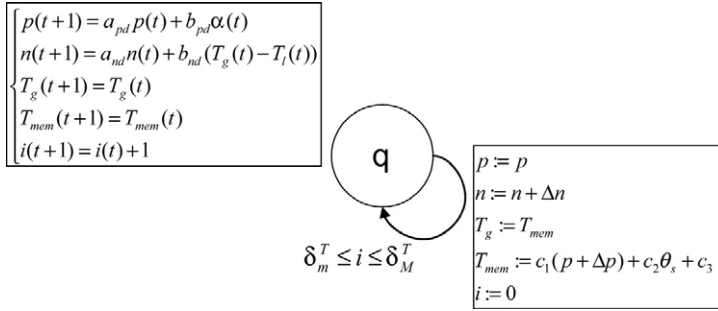


Fig. 4. Discrete-time model $\mathcal{S}_d(T)$ with dwell times and non-synchronization errors.

Remark 19. By comparing the expression for the reset (16) with the expression for the reset R_d in (9), it becomes clear that disturbance d_r living in $I_p \times I_n$ models the non-synchronization between the switching times and the sampling times. Therefore, the set $I_p \times I_n$ models here what the function ρ models in the general procedure of Section 4. Note that the set $I_p \times I_n$ tends to the set $\{0\}$ as the sampling time T goes to 0.

Finally by (10), the minimum and maximum dwell times associated with the discretized model $\mathcal{S}_d(T)$ are given by

$$\delta_m^T = \text{int}(\delta_m/T) - 1 = 13, \quad \delta_M^T = \text{int}(\delta_M/T) + 1 = 17.$$

The resulting discretized model $\mathcal{S}_d(T)$ is shown in Fig. 4. The hybrid model $\mathcal{S}_d(T)$ is characterized by a dynamics affected by an unknown additive disturbance T_l that is bounded in a given set. As pointed out in Remark 17, there is no general result that ensures the existence of a safe set for a switching system subject to continuous disturbances. However, we now show that, for the particular switching system $\mathcal{S}_d(T)$ that we are considering here, there exists a non-empty s -safe set.

Consider the dynamics of the discrete-time system $\mathcal{S}_d(T)$, obtained by neglecting the disturbance torque T_l :

$$x(k + 1) = A_d x(k) + B_d u(k), \tag{17}$$

and the reset function occurring at instant t_{TDC} , obtained by neglecting the synchronization errors $[\Delta p \ \Delta n]'$:

$$x(t_{TDC}^+) = M_r x(t_{TDC}^-) + N_r u_r(t_{TDC}^-) + P_r. \tag{18}$$

The evolution of the system $\mathcal{S}_d(T)$ is governed by N consecutive discrete steps of system (17), with $N \in [\delta_m^T, \delta_M^T]$, followed by the reset function (18). A controlled equilibrium point $x_0 = [p_0 \ n_0 \ T_{g0} \ T_{mem0}]'$ exists for both continuous dynamics and the reset function, where:

$$\begin{cases} p_0 = \frac{-b_{pd}}{a_{pd} - 1} \alpha_0, \\ n_0 = \frac{b_{nd}}{1 - a_{nd}} T_{g0} \\ T_{g0} = T_{mem0}, \\ T_{mem0} = c_1 p_0 + c_2 \theta_{s0} + c_3, \end{cases} \tag{19}$$

for some control values α_0 and θ_{s0} that guarantee $x(k) = x_0, \forall k \geq 0$. The equilibrium point x_0 belongs to the state constraining set Λ . Therefore, by translating the origin of the state space to x_0 , Λ contains the origin in its relative interior, as required for applying the theoretical results of Section 4.

We now introduce the torque disturbance T_l , and the synchronization errors Δp and Δn . The engine speed dynamics is the only one affected by those disturbances, and hence we only need to analyze the evolution of n . Consider a neighborhood \mathcal{B} of the equilibrium point n_0 and suppose that, after a reset, n belongs to \mathcal{B} . Since the engine speed dynamics is asymptotically stable ($|a_{nd}| < 1$), before the next switching, n belongs to a contraction of \mathcal{B} if the torque T_{g0} is applied; after the reset, n belongs to \mathcal{B} , if T_l and the synchronization error are sufficiently small. If the sampling time T is small enough, the synchronization error is small as well (cf. Remark 19). Therefore, a non-empty safe set for $\mathcal{S}_d(T)$ does exist.

Step 2.b of Procedure 16 requires the computation of an s -safe set for the discretized system $\mathcal{S}_d(T)$. Since the discrete state space of $\mathcal{S}_d(T)$ is composed by only one discrete state q , the requested s -safe set is of the form $\{q\} \times \Omega$, where $\Omega \subset \Lambda$. An algorithmic procedure for computing Ω , based on Lemma 10, is shown hereafter, where the following operators are needed:

$$R^{-1}(Y, X) = \{x \in X \mid \exists \theta_s \in [\theta_{smin}, \theta_{smax}] : \forall d_r \in I_p \times I_n, \\ M_r x + N_r \theta_s + G_r d_r + P_r \in Y\};$$

$$\text{Reach}(Y, X) = \{x \in X \mid \exists \alpha \in [0, \alpha_{max}] : \forall T_l \in [0, T_{lmax}], A_d x + B_d \alpha + G_d T_l \in Y\}.$$

Algorithm 20 (Computation of Maximal s -safe Set for $\mathcal{S}_d(T)$).

```

INIT:  $\Omega^0 = \Lambda$ ;
MEM:  $\Omega_{old}^0 = \Omega^0$ ;
 $\Omega^{\delta_M^T} = R^{-1}(\Omega^0, \Lambda)$ ;
if  $\Omega^{\delta_M^T} = \emptyset$  go to STOP-NOK;
j = 1;
while j  $\leq \delta_M^T - \delta_m^T$ 
 $\Omega^{\delta_M^T - j} = \text{Reach}(\Omega^{\delta_M^T - j + 1}, \Omega^{\delta_M^T})$ ;
if  $\Omega^{\delta_M^T - j} = \emptyset$  go to STOP-NOK;
j = j + 1;
end while
while  $\delta_M^T - \delta_m^T < j \leq \delta_M^T$ 
 $\Omega^{\delta_M^T - j} = \text{Reach}(\Omega^{\delta_M^T - j + 1}, \Lambda)$ ;
if  $\Omega^{\delta_M^T - j} = \emptyset$  go to STOP-NOK;
j = j + 1;
end while
if  $\Omega^0 = \Omega_{old}^0$  go to STOP-OK;
else go to MEM;
STOP - NOK: s-safe set does not exist
STOP - OK: Return  $\Omega = \Omega^0$  and  $\Omega^1, \Omega^2, \dots, \Omega^{\delta_M^T}$ 
    
```

The algorithm above outputs a sequence of sets $\{\Omega^i\}_{i=0,1,\dots,\delta_M^T}$ and the requested safe set for $\mathcal{S}_d(T)$ is $\Omega = \Omega^0$.

The state of the controlled system evolves along the sequence of sets:

$$\Omega^0 \xrightarrow{\alpha_0} \Omega^1 \xrightarrow{\alpha_1} \dots \xrightarrow{\alpha_{N-1}} \Omega^N \xrightarrow{\theta_s} \Omega^0 \xrightarrow{\alpha_0} \dots$$

where the label on the arrow represents the particular control used to move in the target set.

The set Λ defined by (14) is a polytope, because of the initial constraint given for the state variables. In particular, constraints on n are part of the specifications of the problem, while the constraints on the other variables are essentially of physical nature: for example, the torque is limited by engine power. The dynamics and the reset function are respectively linear and affine, so that each set Ω^i is a polytope too. Therefore functions $R^{-1}(Y, X)$ and $\text{Reach}(Y, X)$ can be easily implemented by using standard *Fourier Motzkin elimination methods* (e.g. [18]) or the algorithms illustrated in [8]. The proposed procedure converges in a finite number of steps and the s -safe set Ω for the discretized system $\mathcal{S}_d(T)$ is obtained as a result.

Step 3. Given the sequence of sets $\{\Omega^i\}_{i=0 \dots sM^T}$ the controller design deals with the issue of finding a control such that, starting from a state in Ω^i , the set Ω^{i+1} is reached in one step, for any i . Since all sets Ω^i are polytopes, the controller design is simplified. In fact, consider the affine dynamics:

$$x(t + 1) = Ax(t) + Bu(t) + Gd(t) + P,$$

and suppose that

$$\begin{aligned} \Omega^i &= \{x \in \Lambda : W_i x \leq M_i\}, \\ \Omega^{i+1} &= \{x \in \Lambda : W_{i+1} x \leq M_{i+1}\}. \end{aligned}$$

Then the set of *all* inputs that guarantee to reach Ω^{i+1} from a given $x \in \Omega^i$ in one step is

$$U_{\text{safe}}(x) = \{u \in U : (W_{i+1}B)u \leq M_{i+1} - (W_{i+1}A)x - W_{i+1}P - \max_{d \in D}(W_{i+1}Gd)\}.$$

Step 4. From the control strategy determined at Step 3, a T -control strategy for the continuous-time switching system \mathcal{S}_c , is immediately determined.

The simulation results obtained by applying the T -control strategy to \mathcal{S}_c are shown in Fig. 5 and demonstrate how:

- the s -safe set Ω for $\mathcal{S}_d(T)$ is also T -safe for the continuous-time model \mathcal{S}_c (indeed in this case $\mu(\Omega) = 1$);
- the controller found for the discretized system $\mathcal{S}_d(T)$ can be successfully applied to the original continuous-time system \mathcal{S}_c .

6. Concluding remarks

The digital idle speed control problem in automotive design is a challenging control problem that has been the subject of extensive investigation. In this paper, we used hybrid system technology to solve the digital idle speed control problem in automotive design. The digital idle speed control problem consists of maintaining the speed of the engine within a given range in the presence of torque disturbances that model the energy demand on the engine posed by subsystems such as air conditioning that kick in at times that are difficult to predict. We took advantage of the characteristics of the problem to simplify a general hybrid model of the engine that is used to derive the control law. The control problem was recast as a safety control

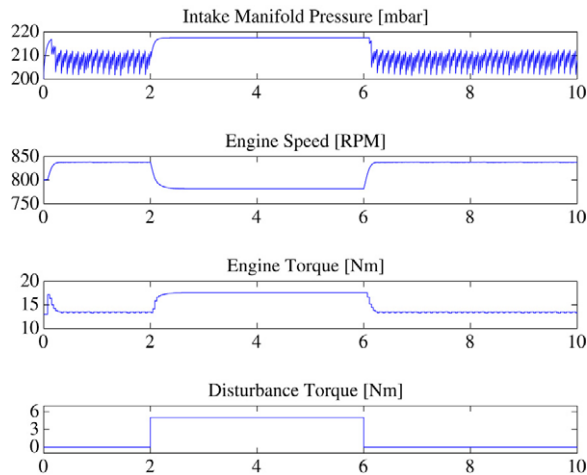


Fig. 5. Simulation of the controlled continuous-time system \mathcal{S}_c .

problem for a hybrid system. Given the characteristics of the idle control problem, we were able to associate with the hybrid model a switching system with minimum and maximum dwell times for which we have been able to derive control strategies with guaranteed properties. These methods, while motivated by the idle control problem, are of general interest for hybrid systems for which minimum and maximum dwell times can be established. The most relevant novelty of our approach is that we do not assume that sampling times and switching times are synchronized. This is an important technical aspect in general, and in particular for our application, where there is no reason why sampling and switching due to reaching a dead center should be synchronized.

Acknowledgement

We would like to thank Dr. Giovanni Girasole for the software implementation of the proposed methodology.

References

- [1] A. Balluchi, M.D. Di Benedetto, C. Pinello, C. Rossi, A.L. Sangiovanni-Vincentelli, Hybrid control in automotive applications: the cut-off control, *Automatica* 35 (March) (1999) 519–535 (Special Issue on Hybrid Systems).
- [2] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, C. Pinello, A.L. Sangiovanni-Vincentelli, Automotive engine control and hybrid systems: Challenges and opportunities, *Proc. IEEE* 88 (2000) 888–912 (Special Issue on Hybrid Systems, Invited Paper).
- [3] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, G. Miconi, U. Pozzi, T. Villa, H. Wong-Toi, A.L. Sangiovanni-Vincentelli, Maximal safe set computation for idle speed control of an automotive engine, in: N. Lynch, B.H. Krogh (Eds.), *Hybrid Systems: Computation and Control*, in: *Lecture Notes in Computer Science*, vol. 1790, Springer-Verlag, New York, USA, 2000, pp. 32–44.
- [4] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, G. Girasole, A.L. Sangiovanni-Vincentelli, Idle speed control design and verification for an automotive engine, in: *Proc. International Workshop on Modeling, Emissions and Control in Automotive Engines, MECA01*, Salerno, Italy, September 2001.
- [5] A. Balluchi, L. Benvenuti, L. Berardi, E. De Santis, M.D. Di Benedetto, G. Girasole, G. Pola, Engine idle speed control via maximal safe-set computation in the crank-angle domain, in: *2002 IEEE International Symposium on Industrial Electronics, IEEE-ISIE 2002*, 8–11 July, L'Aquila, Italy, 2002.
- [6] L. Berardi, E. De Santis, M.D. Di Benedetto, G. Pola, Controlled safe sets for continuous-time linear systems, in: *European Control Conference ECC01*, 4–7 September, Porto, Portugal, 2001, pp. 803–808.

- [7] F. Borrelli, A. Bemporad, M. Fodor, D. Hrovat, in: M.D. Di Benedetto, A.L. Sangiovanni-Vincentelli (Eds.), *A Hybrid Approach to Traction Control*, Hybrid Systems: Computation and Control, in: Lecture Notes in Computer Science, vol. 2034, Springer-Verlag, New York, USA, 2000, pp. 162–174.
- [8] P. D'Alessandro, M. Dalla Mora, E. De Santis, Techniques of linear programming based on the theory of convex cones, *Optimization* 20 (1989) 761–777.
- [9] E. De Santis, M.D. Di Benedetto, L. Berardi, Computation of maximal safe sets for switching systems, *IEEE Trans. Automat. Control* 40 (2) (2004) 184–195.
- [10] E. De Santis, M.D. Di Benedetto, G. Pola, Digital control of continuous-time switching systems with safety constraints, in: *IEEE Conference on Decision and Control 2004, CDC04*, Atlantis, Paradise Island, Bahamas, December 2004, pp. 1878–1883.
- [11] E. De Santis, M.D. Di Benedetto, G. Girasole, Digital idle speed control of automotive engines using hybrid models, in: *IFAC World Congress*, Prague, July 2005.
- [12] E. De Santis, M.D. Di Benedetto, G. Pola, Digital idle speed control of automotive engines: A safety problem for hybrid systems, Technical Report no. R.06-83, Department of Electrical Engineering and Computer Science, University of L'Aquila, Italy, March 2006.
- [13] J.L. Kelley, I. Namioka, *Linear Topological Spaces*, Springer Verlag, 1963.
- [14] D. Liberzon, *Switching in Systems and Control*, Birkhauser, 2003.
- [15] J. Lygeros, C. Tomlin, S. Sastry, Controllers for reachability specifications for hybrid systems, *Automatica* 35 (1999) (Special Issue on Hybrid Systems).
- [16] A.S. Morse, Supervisory control of families of linear set-point controllers — part 1: exact matching, *IEEE Trans. Automat. Control* 41 (10) (1996) 1413–1431.
- [17] C. Özveren, A. Willsky, Observability of discrete event dynamic systems, *IEEE Trans. Automat. Control* 35 (7) (1990) 797–806.
- [18] J. Stoer, C. Witzgall, *Convexity and Optimization in Finite Dimensions I*, Springer-Verlag, Berlin, Heidelberg, New York, 1970.
- [19] Z. Sun, S.S. Ge, *Switched Linear Systems: Control and Design*, Springer, 2005.